CWE

Annex 14.29: extended LTA inclusion
Submitted as part of the CWE Day Ahead FB MC approval package
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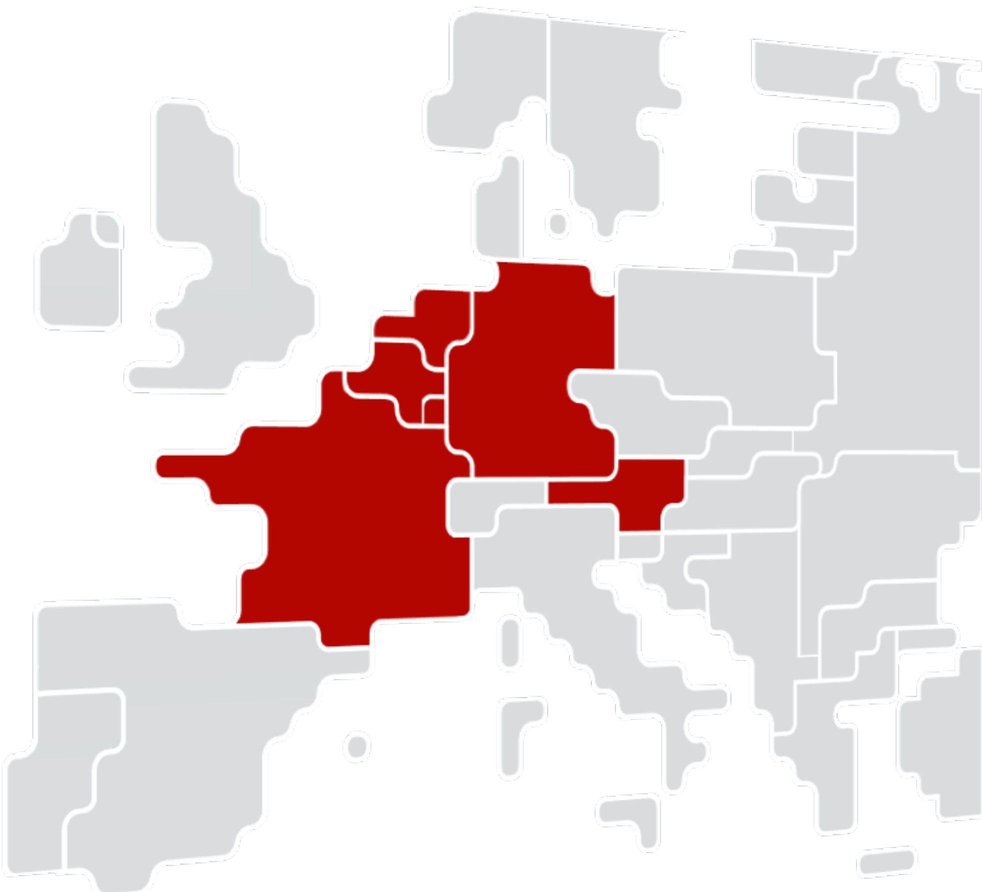

May 2020

Extended formulation for LTA inclusion

Description, shadow prices and price formation

October 2019

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Contents

1 Introduction

We first consider in Section 2 a three nodes flow-based network example to cover concrete aspects of the LTA coverage problem and of the new methodology. The new methodology is based on a standard result to describe the "convex hull of the union of two polyhedra"[1].

The example is used to compare (a) models and market outcomes, (b) shadow prices and price formation, and (c) the congestion rent compared to LTA liabilities. It is shown that formulas relating bidding area market prices to shadow prices of PTDF or LTA constraints are identical to their classic flow-based or ATC counterparts. It is also discussed why the LTA coverage process guarantees to avoid a missing money issue to cover LTA liabilities.

Appendix A then presents the same developments with general notation to describe the approach and related results in full generality.

2 A detailed example

The base case example[2] is first presented, where a missing money problem for LTA liabilities occur. Next Sections then discuss approaches for the LTA coverage process.

2.1 The base case

We consider first a base case example with a three-node network (see figure 1 below) where:

- node A has two supply orders:
  400MWh @ 10€/MWh
  600MWh @ 20€/MWh
- node B has two demand orders:
  100MWh @ 70€/MWh
  900MWh @ 60€/MWh
- node C has one demand order:
  1000MWh @ 50€/MWh

and the following PTDF constraints apply:

$$
\begin{aligned}
&0.75 netpos_b + 0.5 netpos_c \leq 250 \\
&netpos_a \geq -1500
\end{aligned}
$$

Market outcome:

- A exports 450MWh, $price_a = 20$ €$/MWh$
- B imports 100MWh, $price_b = 65$ €$/MWh$
- C imports 350MWh, $price_c = 50$ €$/MWh$
- Welfare = 19 500 €
- Congestion rent $= -450 \times 20 + 100 \times 65 + 350 \times 50 = 15\ 000$€
- Order surpluses = 4 500 €

This market outcome can be obtained by solving the following welfare optimization problem (for each constraint, the associated shadow price variable is indicated on the right in square brackets, to ease later discussions.):

[1] See for example Theorem 1 in [1], or Chapter 2 "Polyhera" in [2]. It is briefly formally described in Appendix.

[2] The base case is an example illustrating flow factor competition in flow-based models presented in *CWE Market Coupling, Flow-Based Forum, Amsterdam, 1st of June 2011, online:* https://www.apxgroup.com/wp-content/uploads/Final_presentation_June_2011.pdf.

$$\max \; welfare := (100)(70)x_{b1} + (900)(60)x_{b2} + (1000)(50)x_c - (400)(10)x_{a1} - (600)(20)x_{a2} \quad (1)$$

$$netpos_a = -400x_{a1} - 600x_{a1} \qquad [price_a = 20] \quad (2)$$
$$netpos_b = 100x_{b1} + 900x_{b2} \qquad [price_b = 65] \quad (3)$$
$$netpos_c = 1000x_c \qquad [price_c = 50] \quad (4)$$
$$0 \leq x_i \leq 1 \qquad \forall i \quad (5)$$

PTDF constraints:

$$0.75netpos_b + 0.5netpos_c \leq 250 \qquad [ShadowPrice_1^{FB} = 60] \quad (6)$$
$$-netpos_a \leq 1500 \qquad [ShadowPrice_2^{FB} = 0] \quad (7)$$

Net exports can be linked to (non-unique) commercial flows:

$$netpos_a = flow_{ba} + flow_{ca} - flow_{ab} - flow_{ac} \qquad [price_a^{FB} = 20] \quad (8)$$
$$netpos_b = flow_{ab} + flow_{ac} - flow_{ba} - flow_{bc} \qquad [price_b^{FB} = 20] \quad (9)$$
$$netpos_c = flow_{ac} + flow_{bc} - flow_{ca} - flow_{cb} \qquad [price_c^{FB} = 20] \quad (10)$$
$$f \geq 0 \quad (11)$$

Note that (8)-(10) imply :

$$netpos_a + netpos_b + netpos_c = 0 \quad (12)$$

- If we assume that the contracted volume of LTA rights is 400 MWh in the direction $A \to B$,

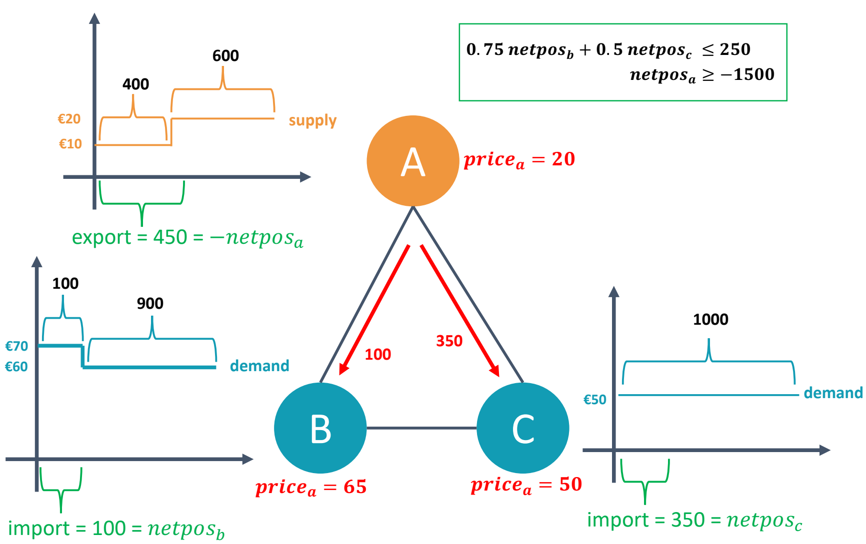


Figure 1: Base case example.

as the price difference is $65 - 20 = 45$, the LTA liabilities are $45 \times 400 = 18000$€.

- However, the congestion rent is equal to: $-450 \times 20 + 350 \times 50 + 100 \times 65 = 15\ 000$€. Hence, the congestion rent does not cover the LTA liabilities of 18 000€.
- This is related to the fact that a solution where A exports to B 400MWh and $netpos_c = 0$, i.e. where $netpos_a = -400, netpos_b = 400, netpos_c = 0$, "commercial flow" $flow_{ab} = 400$ and all other variables are null, is not feasible for the network model: constraint (6) would be violated[3].
- To solve this issue, a preferred solution is to enlarge the FB model "just enough" so as to contain that possibility $netpos_a = -400, netpos_b = 400, netpos_c = 0, flow_{ab} = 400$ (other variables null).
- This is done by considering the smallest network model that can be described by linear constraints of the form $ax \leq b$, which contains both the initial feasible points and the new possibility $netpos_a = -400, netpos_b = 400, flow_{ab} = 400$. Technically, we want the adherence of the convex hull of the union of the initial flow-based domain, and of the LTA domain(the new possibility to add), denoted $\overline{conv}(FB \cup LTA)$.[4]

2.2 The virtual branch approach for $\overline{conv}(FB \cup LTA)$

2.2.1 Model and market outcome

For the base case example above, let FB be the set of feasible net positions and flows described by conditions (6)-(11) and let LTA be the set of net positions that can be obtained by allowing a flow $f_{ab} \in [0; 400], netpos_a = -f_{ab}, netpos_b = f_{ab}, netpos_c = 0$ (and all other flows set to zero). Actually, for LTA, only the extreme case $f_{ab} = 400, netpos_a = -400, netpos_b = 400, netpos_c = 0$ (other flows null) needs to be considered for inclusion in the network model: all intermediate cases with $f_{ab} \in [0; 400]$ will automatically be included as well.

In the Virtual branch approach, one uses new PTDF constraints to describe $\overline{conv}(FB \cup LTA)$. The market outcome obtained is further discussed below and depicted on figure 2. It can be shown that for our example, the new PTDF constraints (18)-(20) together with the system condition $netpos_a + netpos_b + netpos_c = 0$ (here replaced by (21)-(23) as done in the base example above) exactly describes $\overline{conv}(FB \cup LTA)$.

The following small welfare optimization problem hence describes the market clearing problem with the network model enlarged just enough as to *guarantee* that the congestion rent will cover the LTA liabilities.

$$\max\ welfare := (100)(70)x_{b1} + (900)(60)x_{b2} + (1000)(50)x_c - (400)(10)x_{a1} - (600)(20)x_{a2} \quad (13)$$

[3] It is shown below that if this possibility is feasible for the network model, it is guaranteed that the congestion rent covers the LTA liabilities.

[4] Technically, one wants this convex hull "with its boundary included", hence the notation $\overline{conv}$ to distinguish from $conv$, to ensure that the enlarged set is still a polyhedron of the form $\{x|Ax \leq b\}$. This is because if P_1 and P_2 are two polyhedra, i.e. two sets of the form $\{x|Ax \leq b\}$, $conv(P_1 \cup P_2)$ might not be a polyhedron as it might "not contain its boundary", see Appendix C for a small illustrative example and discussion.

$$netpos_a = -400x_{a1} - 600x_{a1} \qquad [price_a = 20] \tag{14}$$
$$netpos_b = 100x_{b1} + 900x_{b2} \qquad [price_b = 63.75] \tag{15}$$
$$netpos_c = 1000x_c \qquad [price_c = 50] \tag{16}$$
$$0 \le x_i \le 1 \qquad \forall i \tag{17}$$

PTDF constraints including (VB):

$$-2netpos_a + netpos_b \le 1200 \qquad [ShadowPrice_1^{FB} = 0] \tag{18}$$
$$-24netpos_a + 11netpos_b \le 14000 \qquad [ShadowPrice_2^{FB} = 1.25] \tag{19}$$
$$-netpos_a \le 1500 \qquad [ShadowPrice_3^{FB} = 0] \tag{20}$$

Net exports can be linked to (non-unique) commercial flows,:

$$netpos_a = flow_{ba} + flow_{ca} - flow_{ab} - flow_{ac} \qquad [price_a^{FB} = 50] \tag{21}$$
$$netpos_b = flow_{ab} + flow_{ac} - flow_{ba} - flow_{bc} \qquad [price_b^{FB} = 50] \tag{22}$$
$$netpos_c = flow_{ac} + flow_{bc} - flow_{ca} - flow_{cb} \qquad [price_c^{FB} = 50] \tag{23}$$
$$f \ge 0 \tag{24}$$

Market outcome for LTA inclusion based on the virtual branch approach:

- A exports 537.5MWh, $price_a = 20$ €$/MWh$
- B imports 100MWh, $price_b = 63.75$ €$/MWh$
- C imports 437.5MWh, $price_c = 50$ €$/MWh$
- Welfare = 22 125 €
- Congestion rent $= -537.5 \times 20 + 100 \times 63.75 + 437.5 \times 50 = 17\ 500$€
- Order surpluses = 4 625 €

2.2.2 Shadow prices and price formation

One can observe the following classical relations:

$$price_l = price^{FB} + \sum_m ptdf_{m,l} ShadowPrice_m \tag{25}$$

where $price^{FB}$ corresponds to the "system price" equal to $50 = price_a^{FB} = price_b^{FB} = price_c^{FB}$ in the example, see (13)-(24). One can derive from these relations:

$$price_k - price_l = \sum_m ShadowPrice_m (ptdf_{m,k} - ptdf_{m,l}) \tag{26}$$

For example, the price difference

$$price_b - price_a = 63.75 - 20 = 43.75 \tag{27}$$

is equal to

$$ShadowPrice_2^{FB}(ptdf_{2,b} - ptdf_{2,a}) = 1.25[11 - (-24)] = 43.75 \tag{28}$$

2.2.3 Congestion rent and LTA coverage

We observe now that the congestion rent covers the LTA liabilities:

- LTA liabilities are given by the contracted volume times the price difference between area B and area A, that is $400 \times (63.75 - 20) = 400 \times 43.75 = 17500$€
- On the other side, the congestion rent is equal also to 17500€, cf. the computation above.

The missing money problem has disappeared.

Let us see on this example the reason why the missing money disappears in general when one considers $\overline{conv}(FB \cup LTA)$ (the statement with more general notation is discussed in Section A.3 below).

This is related to the following: for the market prices $price_a^* = 20, price_b^* = 63.75, price_c^* = 50$ considered as fixed parameters, the operation of the transmission system given by the market out-

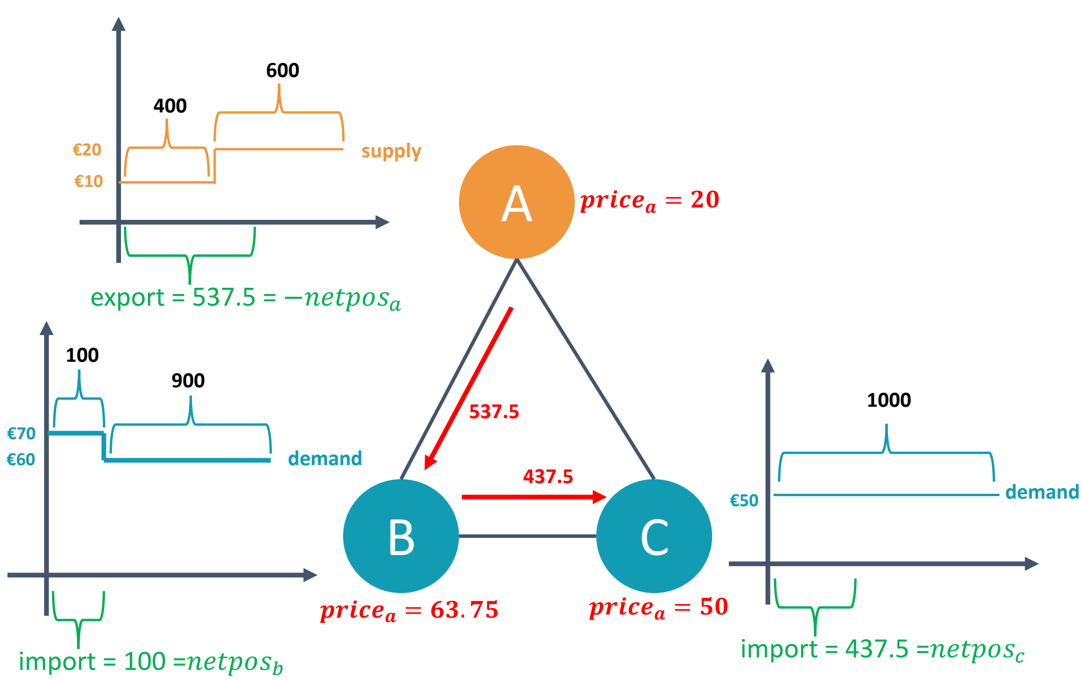


Figure 2: Market outcome with LTA coverage

come $netpos_a = -537.5, netpos_b = 100, netpos_c = 437.5$ is optimal for the following maximization problem:

$$\max congestion\ rent = 20netpos_a + 63.75netpos_b + 50netpos_c \tag{29}$$

subject to the network model constraints (18)-(24)

As the constraints (18)-(24) contain "by construction" the point $netpos_a = -400, netpos_b = 400, netpos_c = 0$, we know that the maximum obtained in (29) is at least equal to $20 \times (-400) + 63.75 \times 400 = 17500$€.

So whatever the solution of the market outcome is, as the net positions will be optimal for (29) (assuming that we know that 20, 63.75 and 50 are the fixed market prices obtained from the market outcome), the congestion rent will be at least 17 500 €. In the present example, using the optimal net positions $netpos_a = -537.5, netpos_b = 100, netpos_c = 437.5$ given by the market outcome, the congestion rent is actually exactly equal to 17 500 €.

2.3 The Extended formulation approach for $\overline{conv}(FB \cup LTA)$

Exactly the same market outcome as with the virtual branch approach in Section 2.2, depicted on figure 2 above, is obtained with the following model based on the new methodology proposed for LTA coverage. The only difference lies in the set of shadow prices "explaining" the (same) bidding area market prices and bidding area price differences.

2.3.1 Model and market outcome

Extended formulation approach:

$$\max\ welfare := (100)(70)x_{b1} + (900)(60)x_{b2} + (1000)(50)x_c - (400)(10)x_{a1} - (600)(20)x_{a2} \tag{30}$$

$$netpos_a = -400x_{a1} - 600x_{a1} \qquad [price_a = 20] \tag{31}$$

$$netpos_b = 100x_{b1} + 900x_{b2} \qquad [price_b = 63.75] \tag{32}$$

$$netpos_c = 1000x_c \qquad [price_c = 50] \tag{33}$$

$$0 \leq x_i \leq 1 \qquad \forall i \tag{34}$$

Virgin PTDF constraints with dedicated net export and flow variables for FB:

$$0.75netpos_b^{FB} + 0.5netpos_c^{FB} \leq \alpha_1 250 \qquad [ShadowPrice_1^{FB} = 55] \tag{35}$$

$$-netpos_a^{FB} \leq \alpha_1 1500 \qquad [ShadowPrice_2^{FB} = 2.5] \tag{36}$$

$$\tag{37}$$

$$netpos_a^{FB} = flow_{ba}^{FB} + flow_{ca}^{FB} - flow_{ab}^{FB} - flow_{ac}^{FB} \qquad [price_a^{FB} = 22.5] \tag{38}$$

$$netpos_b^{FB} = flow_{ab}^{FB} + flow_{ac}^{FB} - flow_{ba}^{FB} - flow_{bc}^{FB} \qquad [price_b^{FB} = 22.5] \tag{39}$$

$$netpos_c^{FB} = flow_{ac}^{FB} + flow_{bc}^{FB} - flow_{ca}^{FB} - flow_{cb}^{FB} \qquad [price_c^{FB} = 22.5] \tag{40}$$

$$flow^{FB} \geq 0 \tag{41}$$

LTA constraints with dedicated net export and flow variables for LTA

$$flow_{ab}^{LTA} \leq \alpha_2 400 \qquad [ShadowPrice_{ab}^{LTA} = 43.75] \tag{42}$$
$$other\ flow\ variables = 0 \tag{43}$$
$$\tag{44}$$
$$netpos_a^{LTA} = flow_{ba}^{LTA} + flow_{ca}^{LTA} - flow_{ab}^{LTA} - flow_{ac}^{LTA} \qquad [price_a^{LTA} = 20] \tag{45}$$
$$netpos_b^{LTA} = flow_{ab}^{LTA} + flow_{ac}^{LTA} - flow_{ba}^{LTA} - flow_{bc}^{LTA} \qquad [price_b^{LTA} = 63.75] \tag{46}$$
$$netpos_c^{LTA} = flow_{ac}^{LTA} + flow_{bc}^{LTA} - flow_{ca}^{LTA} - flow_{cb}^{LTA} \qquad [price_c^{LTA} = 50] \tag{47}$$
$$flow^{LTA} \geq 0 \tag{48}$$

Constraints relating the original net export and flow variables to their duplicates used to describe respectively the virgin flow-based and LTA domains:

$$netpos_i = netpos_i^{FB} + netpos_i^{LTA} \qquad i \in \{a, b, c\} \tag{49}$$
$$flow_{ij} = flow_{ij}^{FB} + flow_{ij}^{LTA} \qquad i, j \in \{a, b, c\} \tag{50}$$
$$\alpha_1 + \alpha_2 = 1 \tag{51}$$
$$\alpha_1, \alpha_2 \geq 0 \tag{52}$$

Market outcome for LTA inclusion based on the extended formulation approach:

- A exports 537.5MWh, $price_a = 20$ €$/MWh$
- B imports 100MWh, $price_b = 63.75$ €$/MWh$
- C imports 437.5MWh, $price_c = 50$ €$/MWh$
- Welfare = 22 125 €
- Congestion rent $= -537.5 \times 20 + 100 \times 63.75 + 437.5 \times 50 = 17\ 500$€
- Order surpluses = 4 625 €

2.3.2 Shadow prices and price formation

Let us first observe that we have the same bidding area market prices but a different set of shadow prices as we now have the virgin flow-based constraints and LTA constraints (somehow "scaled" by the α) to model the network, instead of virtual branches.

However, market price differences are explained by similar relations via the shadow prices of the PTDF constraints involved, namely:

Relation identical to (25):

$$price_l = price^{FB} + \sum_m ptdf_{m,l} ShadowPrice_m \tag{53}$$

where $price^{FB}$ corresponds to the "system price" now equal to $22.5 = price_a^{FB} = price_b^{FB} = price_c^{FB}$ in the example, see (38)-(40).

For example,

$$price_c = 50 = 22.5 + 55(0.5) + 2.5(0) \tag{54}$$

We can then also derive:

Relation identical to (26):

$$price_k - price_l = \sum_m ShadowPrice_m(ptdf_{m,k} - ptdf_{m,l}) \tag{55}$$

For example, the price difference

$$price_b - price_a = 63.75 - 20 = 43.75 \tag{56}$$

is equal to

$$\begin{aligned} &ShadowPrice_1^{FB}(ptdf_{1,b} - ptdf_{1,a}) + ShadowPrice_2^{FB}(ptdf_{2,b} - ptdf_{2,a}) \\ &\qquad = 55[0.75 - 0] + 2.5[0 - (-1)] = 43.75 \end{aligned} \tag{57}$$

2.3.3 Congestion rent and LTA coverage

Discussions regarding the congestion rent would be exactly the same as the discussions in Section 2.2.3 for the virtual branch based approach. Only the constraints of the network model needs to be adapted in the optimization problem for the operation of the transmission system, which is here:

$$\max congestion\ rent = 20netpos_a + 63.75netpos_b + 50netpos_c \tag{58}$$

subject to the network model constraints (35)-(52)

(Conditions (35)-(52)have replaced conditions (18)-(24).)

3 Observations and practical questions for implementation in production

3.1 Assessment of performance gains

Let us first emphase that the new extended formulation methodology presented on an example in Section 2.3 is much more scalable than the approach based on virtual branches: in the extended formulation approach, the number of constraints required is the number of virgin PTDF constraints plus the number of LTA capacity constraints and a number of constraints directly proportional to the number of bidding areas and lines (to link the "duplicate" net position and flow variables for respectively the virgin flow-based and LTA domains to their counterpart lying in the convex hull of the union of these domains).

On the other hand, the virtual branch approach requires to pre-compute a number of PTDF constraints which becomes quickly very large with an increased number of bidding areas. Also, the computation of virtual branches becomes itself very challenging.

The added value of the new methodology is hence two-folds:

- For network models with many more bidding areas as for CORE Region, it makes it tractable to consider the "tight version" of a LTA coverage process (enlarging the original PTDF domain *"just enough"*).
- For network models with a few more bidding areas as required in some Evolved Flow-Based modeling applications, it allows to greatly reduce the number of constraints required to describe the enlarged domain for LTA coverage, with a substantially positive impact on Euphemia performances.

The following figure 3 shows performance gains with preliminary performance tests for Evolved Flow-Based with 7 CWE bidding areas (instead of 5) to model the inclusion of the Alegro HVDC inter-connector, for flow-based plain:

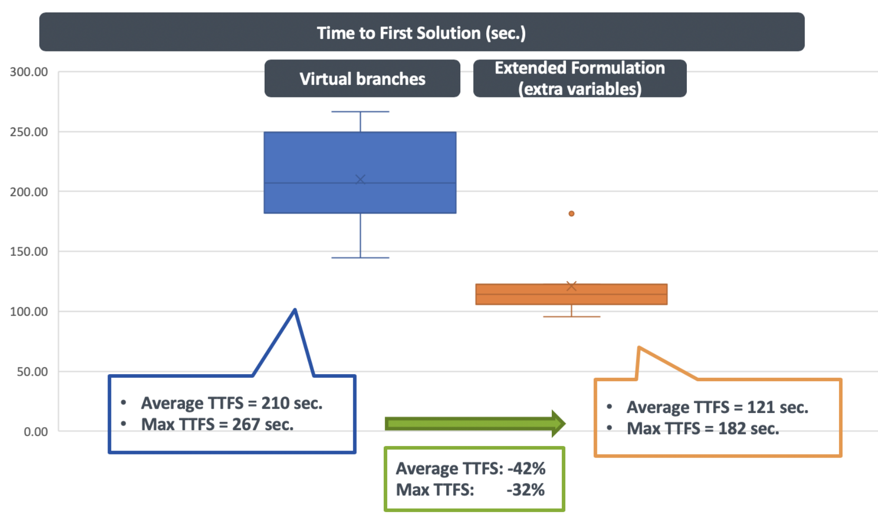


Figure 3: Comparison of Times to First Solution (in seconds), based on Euphemia 10.4, 7 sessions in November 2018 to April 2019 with inclusion of the Alegro HVDC inter-connector, Flow-Based Plain (FBP)

3.2 Interactions with bidding zones outside the balancing area

The modeling alternative only concerns the CWE balancing and the corresponding "net exports" (net positions on the balancing area). It has no impact on other bidding areas such as neighbouring bidding areas connected via ATC lines.

3.3 Interface and PMB changes

For the current prototype implementation, minor changes are needed in the Euphemia input interface:

- Columns LTA_DOWN and LTA_UP should be added to table LINE_CAPACITIES (alternatively, columns CAPACITY_UP and CAPACITY_DOWN could be used as currently no capacity is otherwise specified for "TOP" lines of the CWE area).
- A column (e.g. ISVIRGIN) has to be added to table SESSION_BALANCINGAREAS to indicate if LTAs are to be taken into consideration for the balancing area (note that if the parameter indicates that LTAs should be considered but an empty LTA domain is provided, the behavior is similar to the case where no LTAs should be taken into consideration, although extra variables for LTAs not needed in that case would be created).

Note that the virgin PTDF data is provided via the usual tables for PTDF constraints.

To use the feature, the user has to provide a non-empty LTA domain, via table LINE_CAPACITIES, and specify a virgin domain is used via table SESSION_BALANCINGAREAS. If the LTA domain is empty, the behavior is similar to the case where classical PTDF constraints are provided.

Regarding the Euphemia output interface, shadow prices of the virgin flow-based constraints can be reported as the classical shadow prices of input PTDF constraints. The only new information of interest would be the shadow prices of constraints of the LTA domain. These shadow prices directly explain bidding area price differences as in a classic ATC network model (if a "LTA flow" occurs between bidding areas with different prices, the shadow price of the LTA capacity is equal to the price difference, and the "LTA flow" goes from the cheaper area to the more expensive area), cf. the example discussed in Section 2 and also price conditions derived in Appendix A.2. Note that the price difference between bidding areas can also be explained via the shadow prices of the virgin PTDF constraints, cf. again Section 2 and the example discussed therein.

3.4 Interaction with the intuitiveness patch

Intuitiveness is now enforced via the virgin flow-based domain which might be more restrictive than when the additional conditions to enforce intuitiveness are related to the enlarged flow-based domain $\overline{conv}(FB \cup LTA)$.

Although this requires further assessment and would require advanced technical work, it should be possible to circumvent this by generating those additional conditions to enforce intuitiveness which are related to the enlarged flow-based domain $\overline{conv}(FB \cup LTA)$.

3.5 Impact on the adequacy patch

The behavior of the adequacy patch is not impacted by the reformulation of the LTA coverage: the original net position and flow variables $netpos, flow$ are still available and used for related computations.

The only impact on curtailment aspects could in theory come from restrictions of additional transmission capacities due to intuitiveness being enforced on the virgin flow-based domain which is more restrictive than enforcing intuitiveness on the enlarged domain after LTA coverage, but this impact is not specifically related to the adequacy patch.

3.6 Validation of the implementation

To validate the implementation, it is recommended to design various functional tests based on virtual branches exactly describing $\overline{conv}(FB \cup LTA)$, so that market outcome results computed by Euphemia with virtual branches can be directly compared to market results obtained with the new methodology which has been prototyped in Euphemia. As discussed in the example above, with flow-based plain, market results must be identical. With flow-based in intuitive mode, market results can differ due to the way intuitiveness is, for now, enforced in the new methodology.

A Extended formulation for LTA inclusion in general

The extended formulation is described here with general notation for sessions with classical step bid curves. How the formulation is obtained is briefly discussed in Appendix B.

A.1 Welfare maximization model

$$\max \sum_i Q^i P^i x_i \tag{59}$$

s.t.

$$\sum_{i \in Orders(l)} Q^i x_i = netpos_l \qquad [price_l] \quad \forall l \in Locations \tag{60}$$

$$0 \le x_i \le 1 \qquad \forall i \tag{61}$$

$$\tag{62}$$

$$netpos_l = \widetilde{netpos}_l^{FB} + \widetilde{netpos}_l^{LTA} \qquad [\widetilde{price_l}] \tag{63}$$

$$flow_{l,k} = \widetilde{flow}_{l,k}^{FB} + \widetilde{flow}_{l,k}^{LTA} \tag{64}$$

$$\alpha_1 + \alpha_2 = 1 \qquad [\eta] \tag{65}$$

$$\alpha \ge 0 \tag{66}$$

$$\sum_l ptdf_{m,l} \widetilde{netpos}_l^{FB} \le \alpha_1 \; RAM_m \qquad [ShadowPrice_m] \forall m \tag{67}$$

$$\widetilde{netpos}_l^{FB} = \sum_{k \ne l} \widetilde{flow}_{k,l}^{FB} - \widetilde{flow}_{l,k}^{FB} \qquad [price_l^{FB}] \; \forall l \in Locations \tag{68}$$

$$\widetilde{flow}^{FB} \ge 0 \tag{69}$$

$$\widetilde{flow}_{l,k}^{LTA} \le \alpha_2 \; capacity_{l,k}^{LTA} \qquad [w_{l,k}^{LTA}] \qquad \forall l, k \in Locations \tag{70}$$

$$\widetilde{netpos}_l^{LTA} = \sum_{k \ne l} \widetilde{flow}_{k,l}^{LTA} - \widetilde{flow}_{l,k}^{LTA} \qquad [price_l^{LTA}] \qquad \forall l \in Locations \tag{71}$$

$$\widetilde{flow}^{LTA} \ge 0 \tag{72}$$

A.2 Shadow prices and price formation

Conditions dual to the variables $netpos_l$:

$$\widetilde{price}_l = price_l \tag{73}$$

Next conditions are written by taking (73) into account and replacing $\widetilde{price_l}$ with $price_l$.

Conditions dual to the variables $netpos_l^{FB}$:

$$price_l = price_l^{FB} + \sum_m ptdf_{m,l} ShadowPrice_m \tag{74}$$

Conditions dual to the variables $netpos_l^{LTA}$:

$$\boxed{price_l = price_l^{LTA}} \tag{75}$$

Conditions dual to the variables $flow_{l,k}^{LTA} \geq 0$:

$$w_{l,k}^{LTA} \geq price_k^{LTA} - price_l^{LTA} \tag{76}$$

Using the associated complementary condition $flow_{l,k}^{LTA}(w_{l,k}^{LTA} - price_k^{LTA} + price_l^{LTA}) = 0$, we have:

$$\boxed{flow_{l,k}^{LTA} > 0 \Rightarrow price_k^{LTA} - price_l^{LTA} = w_{l,k}^{LTA} \geq 0} \tag{77}$$

Conditions dual to the variables $f_{l,k}^{FB} \geq 0$:

$$0 \geq price_k^{FB} - price_l^{FB} \tag{78}$$

Considering (78) for all pairs l, k gives $price_k^{FB} = price_l^{FB}$ (assuming that locations form a connected component), and conditions (74) can be rewritten as:

$$\boxed{price_l = price^{FB} + \sum_m ptdf_{m,l} ShadowPrice_m,} \tag{79}$$

where $price^{FB}$ corresponds to the "system price".

We then have the usual relations:

$$\boxed{price_k - price_l = \sum_m ShadowPrice_m (ptdf_{m,k} - ptdf_{m,l})} \tag{80}$$

A.3 Congestion rent and LTA coverage

We show here with general notation that with the LTA coverage methodology, the congestion rent is always sufficient to cover LTA liabilities.

Let us consider an optimal solution $(x, netpos, flow)$ to the welfare maximization problem (59)-(72) and consider the market prices $price_l^*$ obtained as optimal dual variables of (60).

We want to prove the following inequality:

$$\sum_l netpos_l^* \ price_l^* \geq \sum_{l,k} (price_k^* - price_l^*)^+ capacity_{l,k}^{LTA} \tag{81}$$

The congestion rent appears on the left-hand side of the inequality and is expressed as the revenue from operating the transmission network, i.e. realized from the buy/sell operations: $netpos_l^* > 0$

if a volume is sold by the operator to location l, $netpos_l^* < 0$ if a volume is bought from location l and the left-hand side represents the sum of the associated money transfers given the locational market prices $price_l^*$.

The right-hand side represents LTA liabilities. The notation $(price_k^* - price_l^*)^+$ means the price in location k minus the price in location l counted only of the price is higher in location k, and 0 otherwise ($(price_k^* - price_l^*)^+$ is hence non-null only if the price in location k is higher than in location l, in which case $(price_k^* - price_l^*)^+$ is equal to that price difference). Multiplying $(price_k^* - price_l^*)^+$ by the volume of LTA rights in the direction $l \to k$ denoted by $capacity_{l,k}^{LTA}$, and summing up over all possibilities for l, k, provides the total LTA liabilities.

To prove (81), we will use the fact that $(netpos^*, flow^*)$ obtained from the market clearing process solves the following profit maximization problem for the transmission system, where locational market prices $price^*$ are fixed parameters and where the operator seeks to find best import/export decisions $(netpos, flow)$ given those prices and assuming "infinite market depth" (i.e. without worrying about the order books):

$$\max_{(netpos, flow)} \sum_l netpos_l \; price_l^* \tag{82}$$

s.t. to network constraints (63)-(72).

To prove the inequality (81), it is hence sufficient to find a solution $(\overline{netpos}, \overline{flow})$ feasible for (63)-(72) such that:

$$\sum_l \overline{netpos}_l \; price_l^* \geq \sum_{l,k} (price_k^* - price_l^*)^+ capacity_{l,k}^{LTA}, \tag{83}$$

as the congestion rent is at least as high as the left-hand side.

Such a feasible solution can be straightforwardly constructed as the network model has been enlarged for that purpose, and can be given by:

$$
\begin{aligned}
& flow_{l,k} = flow_{l,k}^{LTA} := capacity_{l,k}^{LTA} && if \quad price_k^* > price_l^* \\
& flow_{l,k} = flow_{l,k}^{LTA} := 0 && if \quad price_k^* \leq price_l^* \\
& netpos_l = netpos_l^{LTA} := \sum_{k \neq l} (flow_{k,l} - flow_{l,k}) = \sum_{k \neq l} (flow_{k,l}^{LTA} - flow_{l,k}^{LTA}) && \\
& \alpha_{LTA} = 1 && \\
& \alpha_{FB} := 0, netpos_l^{FB} := 0, flow_{l,k}^{FB} := 0 &&
\end{aligned}
$$

B From convex combinations to the extended formulation for $\overline{conv}(FB \cup LTA)$

A convex combination of two points x, y is the set of points that can be written as $\alpha_1 x + \alpha_2 y$ with $\alpha_1 + \alpha_2 = 1, \alpha_1 \geq 0, \alpha_2 \geq 0$.

It is the goal of the constraints (87)-(96) where one makes a convex combination of a point $(netpos^{FB}, flow^{FB})$ in the flow-based domain described by conditions (91)-(93) and a point in the LTA domain described by conditions (94)-(96). These constraints (87)-(96) are then transformed via a substitution of variables into the formulation (63)-(72). Details follow in next paragraphs, where cases with $\alpha_1 = 0$ or $\alpha_2 = 0$ are also discussed.

For $\alpha_1 > 0, \alpha_2 > 0, \alpha_1 + \alpha_2 = 1$, constraints (91)-(93) are fully equivalent to the original flow based constraints (just multiplied by a strictly positive number α_1) and constraints (94)-(96) are similarly fully equivalent to the original LTA constraints.

Making the substitution $\widetilde{netpos^{FB}} := \alpha_1 netpos^{FB}, \widetilde{flow^{FB}} := \alpha_1 flow^{FB}, \widetilde{netpos^{LTA}} := \alpha_2 netpos^{LTA}, \widetilde{flow^{LTA}} := \alpha_2 flow^{LTA}$ then exactly provides the extended formulation (63)-(72) in Appendix A.

Let us check that this formulation (63)-(72) also works when $\alpha_1 = 0$ or $\alpha_2 = 0$.

If $\alpha_1 = 0, \alpha_2 = 1$, assuming the PTDF polyhedron is bounded[5], only the solution $\widetilde{netpos^{FB}} = 0, \widetilde{flow^{FB}} = 0$ is feasible for (67)-(69) and we actually pick-up a point in the LTA domain, as constraints (70)-(72) with $\alpha_2 = 1$ are the original LTA constraints. Similarly, if $\alpha_1 = 1, \alpha_2 = 0$, it implies that $\widetilde{netpos^{LTA}} = 0, \widetilde{flow^{LTA}} = 0$ and we actually pick up a point in the flow-based domain, as constraints (67)-(69) with $\alpha_1 = 1$ are the original flow-based constraints.

$$\max \sum_i Q^i P^i x_i \tag{84}$$

$$\sum_{i \in Orders(l)} Q^i x_i = netpos_l \tag{85}$$

$$0 \leq x_i \leq 1 \qquad \forall i \tag{86}$$

$$netpos_l = \alpha_1 \; netpos_l^{FB} + \alpha_2 \; netpos_l^{LTA} \tag{87}$$

$$flow_{l,k} = \alpha_1 \; flow_{l,k}^{FB} + \alpha_2 \; flow_{l,k}^{LTA} \tag{88}$$

$$\alpha_1 + \alpha_2 = 1 \tag{89}$$

$$\alpha \geq 0 \tag{90}$$

$$\alpha_1 \sum_l ptdf_{m,l} \; netpos_l^{FB} \leq \alpha_1 RAM_m \tag{91}$$

$$\alpha_1 netpos_l^{FB} = \alpha_1 \sum_{k \neq l} (flow_{k,l}^{FB} - flow_{l,k}^{FB}) \tag{92}$$

$$\alpha_1 flow^{FB} \geq 0 \tag{93}$$

$$\alpha_2 flow_{l,k}^{LTA} \leq \alpha_2 capacity_{l,k}^{LTA} \tag{94}$$

$$\alpha_2 netpos_l^{LTA} = \alpha_2 \sum_{k \neq l} (flow_{k,l}^{LTA} - flow_{l,k}^{LTA}) \tag{95}$$

$$\alpha_2 flow^{LTA} \geq 0 \tag{96}$$

[5] We refer to the reference [2] for the general case where polyhedra P_1, P_2 involved in the present method to describe $\overline{conv}(P_1 \cup P_2)$ are unbounded.

C An example illustrating the difference between $conv(P_1 \cup P_2)$ and $\overline{conv}(P_1 \cup P_2)$

Consider:

- $P_1 = \{(x, y)|y = 1\}$, the blue line in the figure 4 below,
- $P_2 = \{(x, y)|x = 2, y = 2\}$, the red point in the figure 4 below.

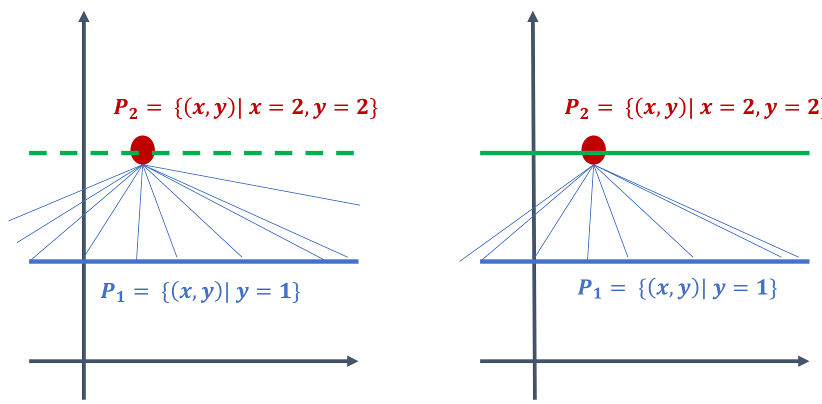


Figure 4: Difference between $conv(P_1 \cup P_2)$ and $\overline{conv}(P_1 \cup P_2)$.

One can check that all the possible convex combinations of the red point P_2 and a point in the blue line P_1 are all the points between the blue line included, and the dotted green line excluded, plus the red point: the dotted green line is a part of the boundary which is not included in "$conv(P_1 \cup P_2)$". Formally:

$$conv(P_1 \cup P_2) = \{(x, y)|(y \geq 1, y{<}2)\} \cup \{(x = 2, y = 2)\} \tag{97}$$

This set cannot be described as a polyhedron, i.e. via non-strict linear inequalities.

However, if we include the boundary green line, cf. the right-hand side of figure 4, i.e. we consider "$conv(P_1 \cup P_2)$ plus its boundary included", which is written $\overline{conv}(P_1 \cup P_2)$, one has:

$$\overline{conv}(P_1 \cup P_2) = \{(x, y)|\ y \geq 1, y \leq 2\} \tag{98}$$

which is now a polyhedron.

For applications in optimization, one needs to work with the second option (otherwise, maximum or minimum might not exist).

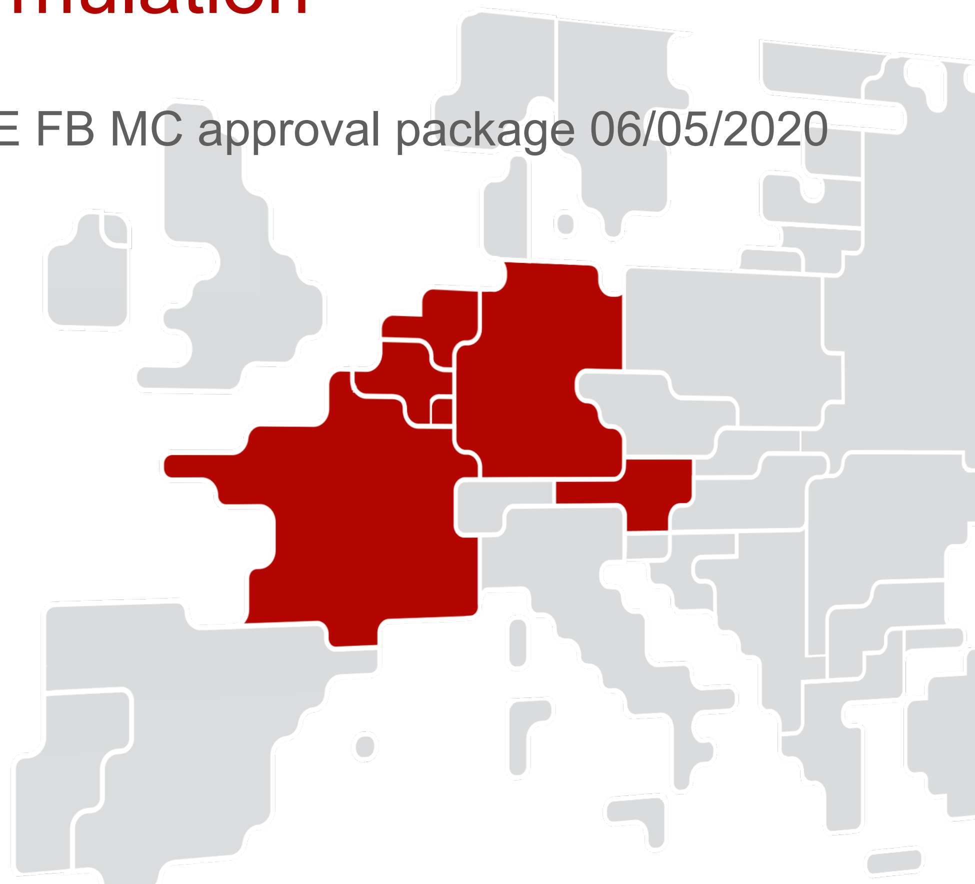
D Bibliography

[1] Michele Conforti, Marco Di Summa, and Yuri Faenza. Balas formulation for the union of polytopes is optimal. *Mathematical Programming*, pages 1–16, 2017.

[2] Giuseppe Lancia and Paolo Serafini. *Compact extended linear programming models*. Springer, 2018.

Annex 14.30 - Pedagogical information on Extended LTA formulation

Submitted as part of the CWE FB MC approval package 06/05/2020



About this document

This document explains the methodologies used for LTA inclusion. This document aims to provide an overview of the changes that are expected considering the performance upgrades for CWE. It consists of the following:

- Reminder of current approach
- Improved virtual branches
- Extended LTA

By means of a high-level introduction, the performance mitigations will significantly reduce the amount of constraints needed for the capacity calculation. This in turn improves performance on both capacity calculation ad allocation side. The improved performance allows the market coupling algorithm to find more optimal solutions, possibly increasing the social welfare.

The implementation of the performance upgrades are expected in Q4 2020

- Improved virtual branches can be expected as of ALEGrO go-live
- Extended LTA with a new release of the market coupling algorithm around Q4 2020

Updates to include changes related to performance mitigations:

- As informed during the CCG of 03/04, performance challenges of FBCE & Euphemia with additional borders became apparent with the preparations for the implementation of ALEGrO: there was lack of scalability of the IT system.
    - The issue: the number of virtual branches which are created for the LTA inclusion process is exponentially linked to the mathematical dimensionality.

Current process

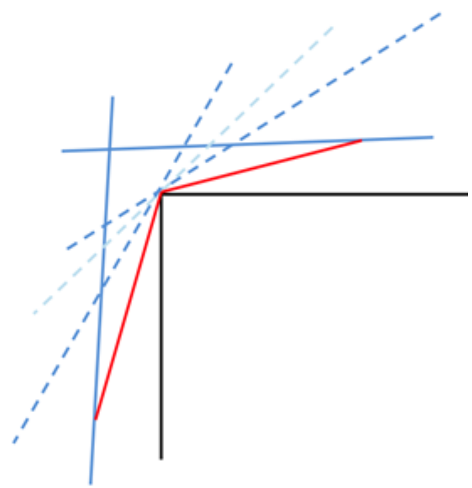
- Currently each real branch is replaced by its virtual counterpart in case the LTA corner creates an overload– leading to a high number of duplicate virtual branches. Hence, the number of virtual constraints generated during the LTA inclusion can be significantly reduced by removing this duplication step
- With the current approach, each virtual branch will be scaled with the corresponding RAM. It will lead to non-physical PTDFs for the virtual branches.
- A detailed example can be found in the annexes of the slides.

TSOs accordingly developed updates of the methodology to perform the LTA inclusion in order to improve performance and the reasons why both are needed:

- Improved virtual branches – Reduction of virtual constraints
- Extended LTA inclusion – Novel way of doing LTA inclusion in the Market Allocation part

Improved virtual branches

- CWE TSOs developed the Improved virtual branches in order to reduce the number of virtual branches. Improved Virtual Branches will not create duplicate as it was the case in the current LTA inclusion.
- Detailed example can be found in annex.
- The Virtual branches will not be scaled with the RAM like in the past.
- For the sake of transparency,
  - The most limiting line (Highest LTA margin) will give its name to the newly constructed branches (virtual branches).
  - The other lines will be shifted similarly to an application of FAV while with the current approach they were deleted.
- Moment of application: ALEGrO technical go-live



ALEGrO technical go-live will happen a bit prior the commercial go-live with ALEGrO constrained to 0.

Update CWE methodologies for CEP and ALEGrO implementation

Extended LTA inclusion

An R&D track with N-SIDE under SDAC governance successfully elaborated an alternative way for LTA inclusion directly in Euphemia.

- In the extended LTA inclusion process, the market coupling algorithm now receives two domains (one FB domain with MinRAM and one LTA domain) representing the flow-based Capacities of the CWE region.
- Euphemia does not recalculate the flow-based domain (it does not create an LTA included domain). Instead, the "Balas formulation for LTA inclusion" allows Euphemia to choose which combination of both domains creates most social welfare, where the share of the LTA domain and the share of virgin FB domain is expressed with a factor alpha.
- Mathematical formulation can be found in *Annex 14.29 – Extended LTA formulation*

Extensive analysis has shown that the implementation of the 'Balas formulation' (Extended LTA inclusion) **yields the same market coupling results** as LTA inclusion via improved virtual branches:

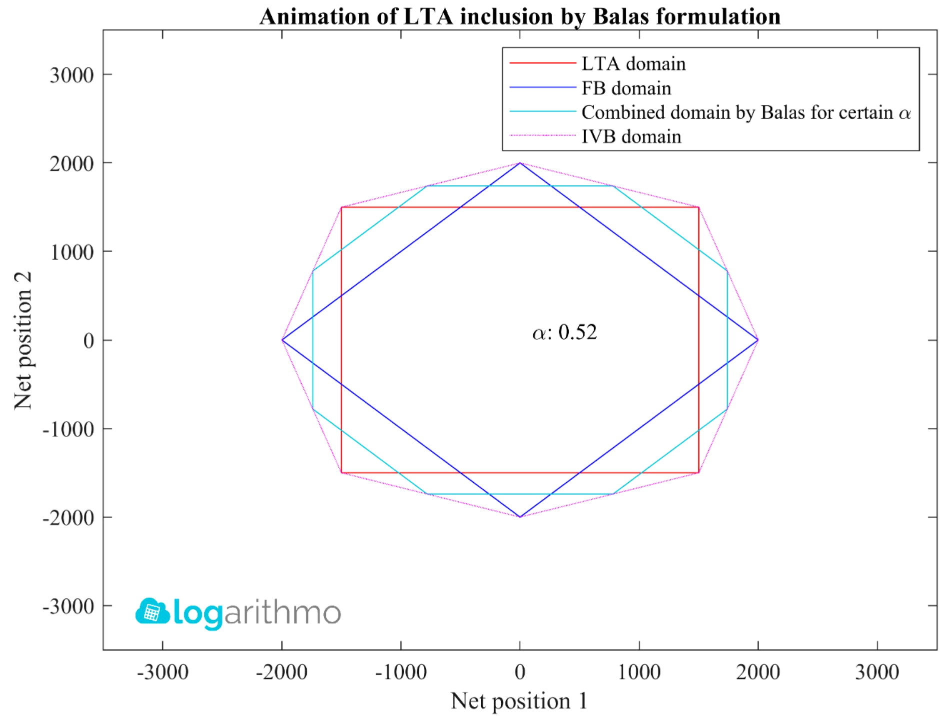
- The min max net position of the FB domain correspond
- In addition, running market coupling simulations generated comparable welfare
- Minor differences have been observed due to rounding and because the 'Balas formulation' is more precise than the virtual branch creation
- Summary of the analysis can be found in annex.

It has been demonstrated that the Extended LTA formulation delivers significant gain in the performance of Euphemia.

- Extended LTA will be used from the introduction of Euphemia 10.5 onwards.
- Even after the switch towards Extended LTA, the improved Virtual Branches Process will be kept for transparency (Final Flow-Based domain), ID ATC extraction, ATC extraction SA … *Improved Virtual branches is not scalable to a large number of BZs (e.g. Core)*

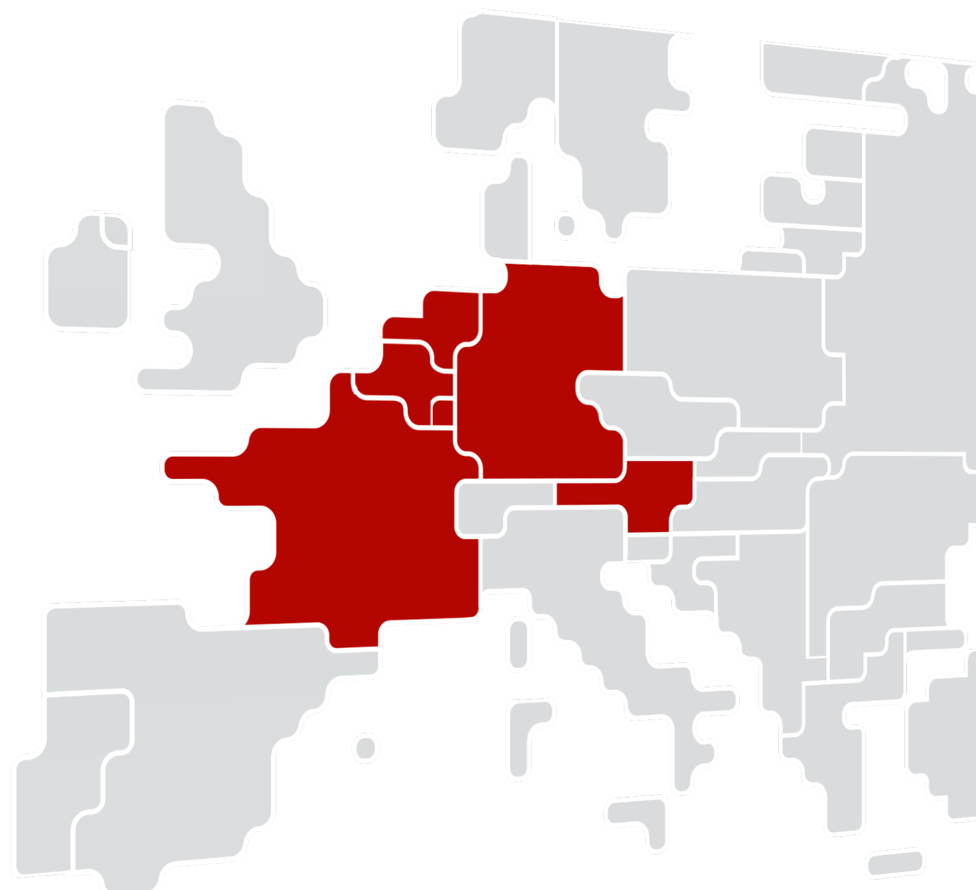
Video exemplifying the extended LTA inclusion method

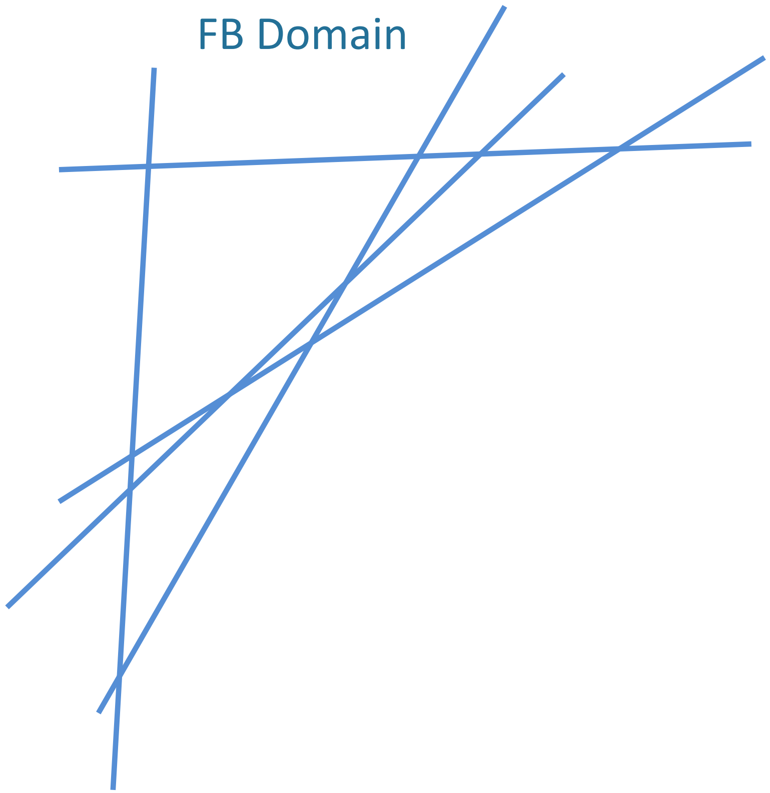
- https://fileserver.logarithmo.de/s/dJNRbzsDQjcKLi6

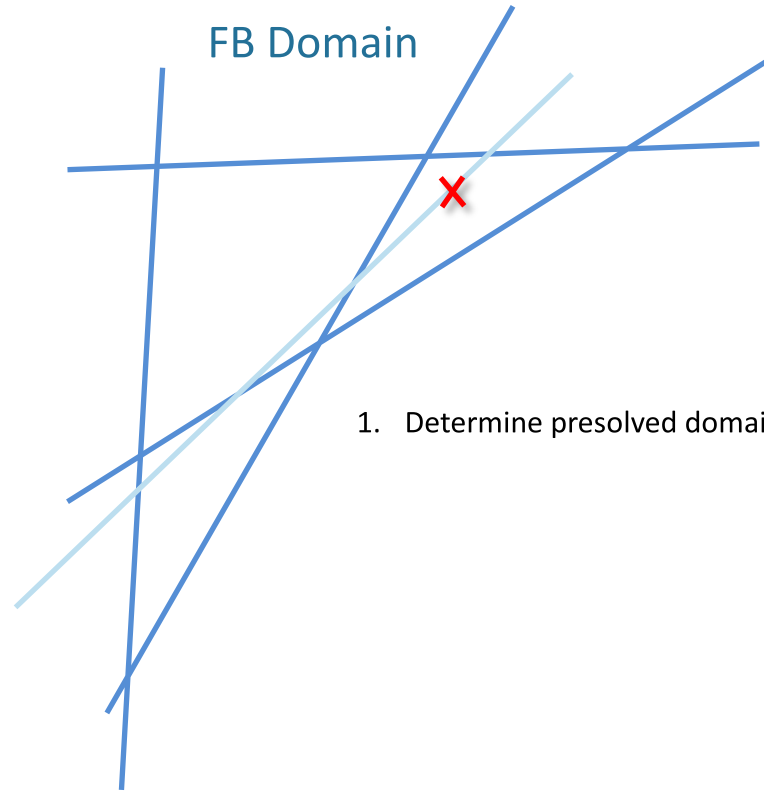


- The current link shows an animation of how extended LTA inclusion works. You can find in blue and in red, the FB domain and the LTA domain respectively which would be given as inputs to the market coupling as explained in the previous slide. The pink domain represents the domain created thanks to LTA inclusion done with virtual branches.
- As you can see in the animation, the cyan domain is the linear combination of the blue and the red domain considering different values of alpha (LTA - alpha=1 & FB - alpha=0). The animation shows that the set of feasible market coupling points will be the same as for the Virtual Branches approach.

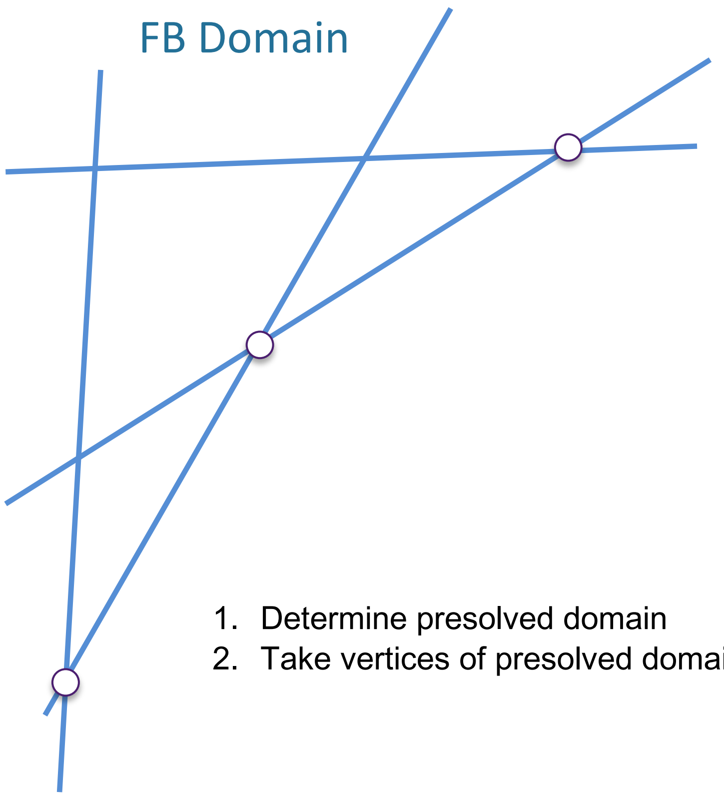
Annex – Current LTA



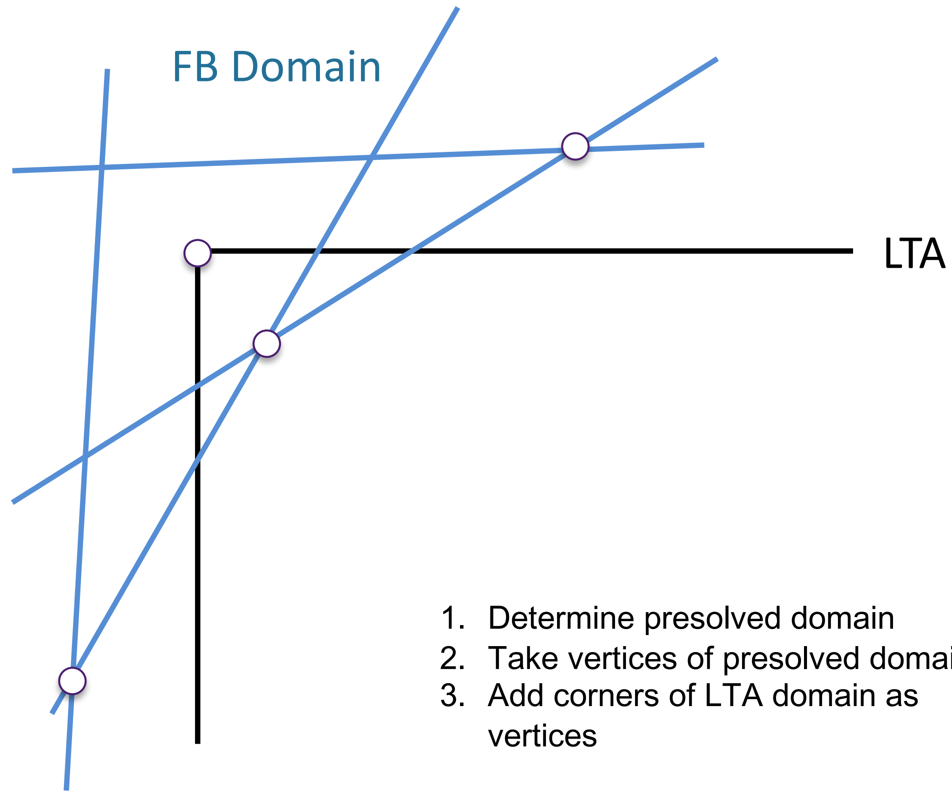




1. Determine presolved domain

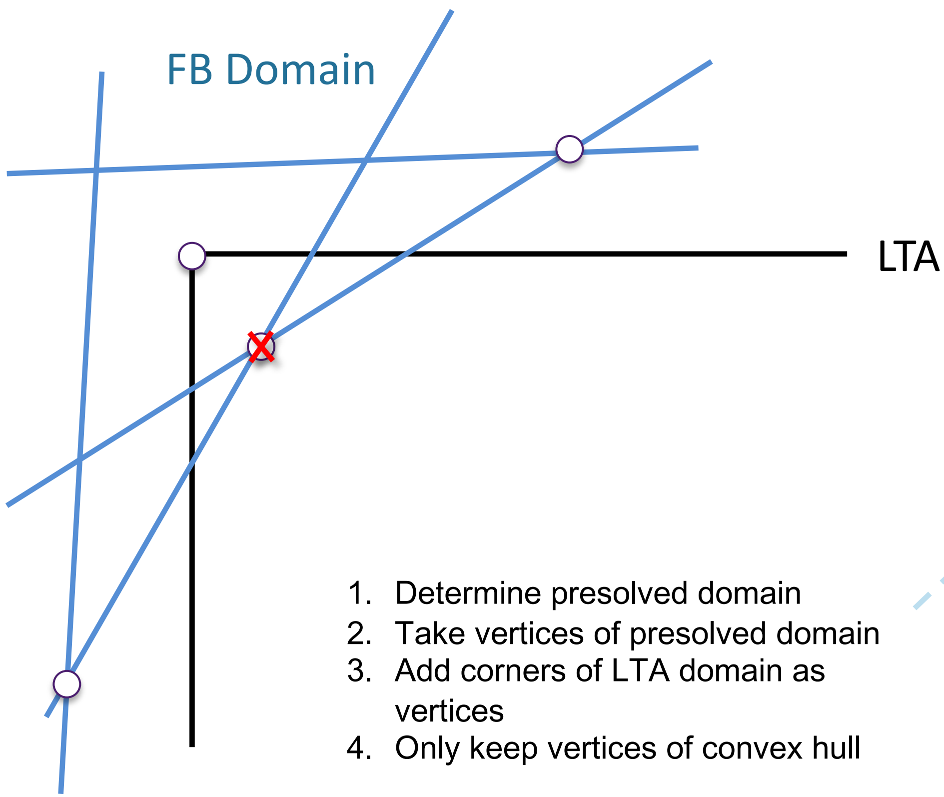


1. Determine presolved domain
2. Take vertices of presolved domain

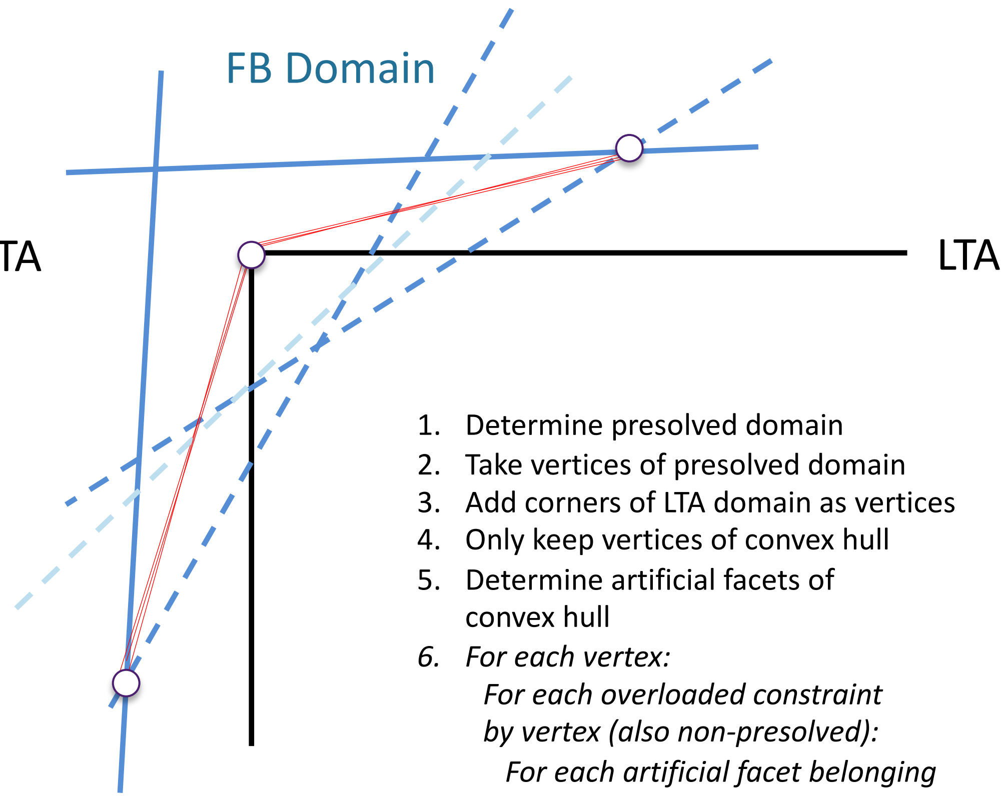


1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices

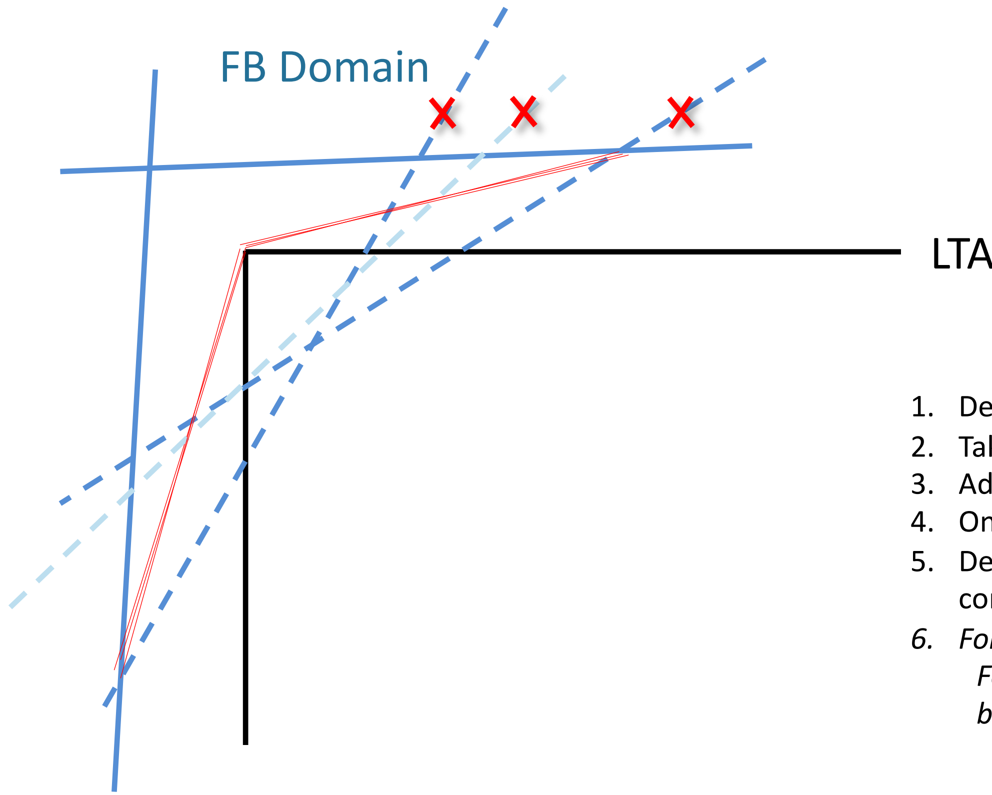
Introduction and Current approach



1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull

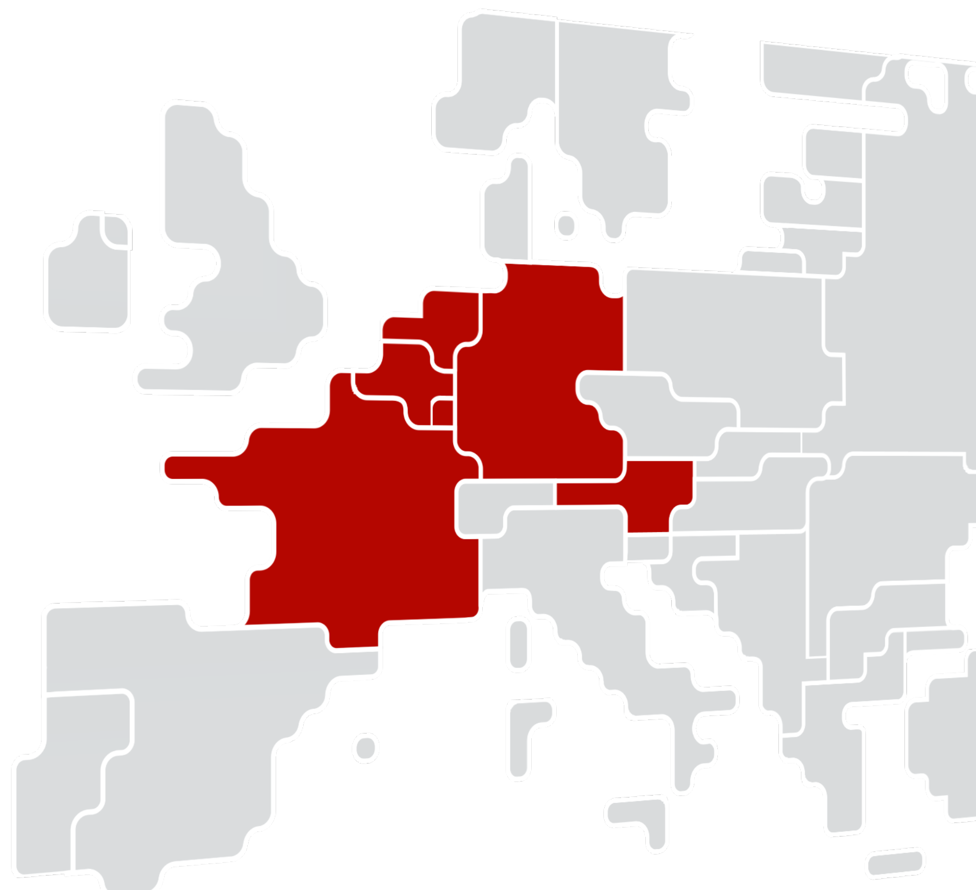


1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull
5. Determine artificial facets of convex hull
6. *For each vertex:*
   For each overloaded constraint by vertex (also non-presolved):
   For each artificial facet belonging to vertex:
   Add a virtual branch based on artificial facet but scaled to constraints desired RAM

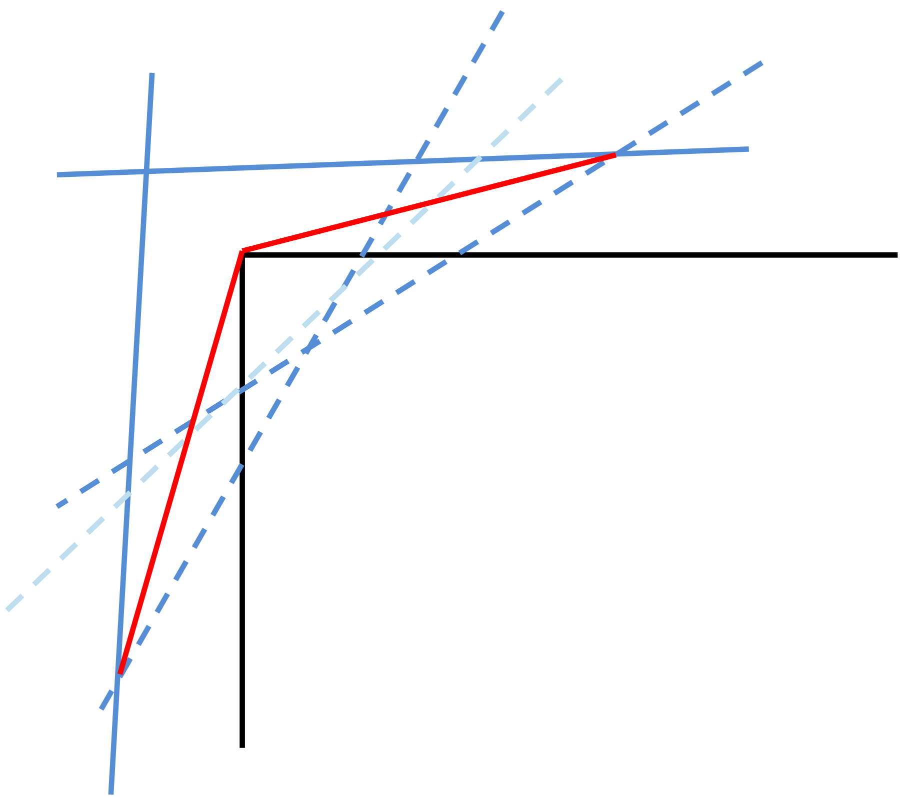


1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull
5. Determine artificial facets of convex hull
6. *For each vertex:*
   For each overloaded constraint by vertex (also non-presolved):
   For each artificial facet belonging to vertex:
   Add a virtual branch based on artificial facet but scaled to constraints desired RAM
7. Delete overloaded constraints

Annex – Improved Virtual Branches

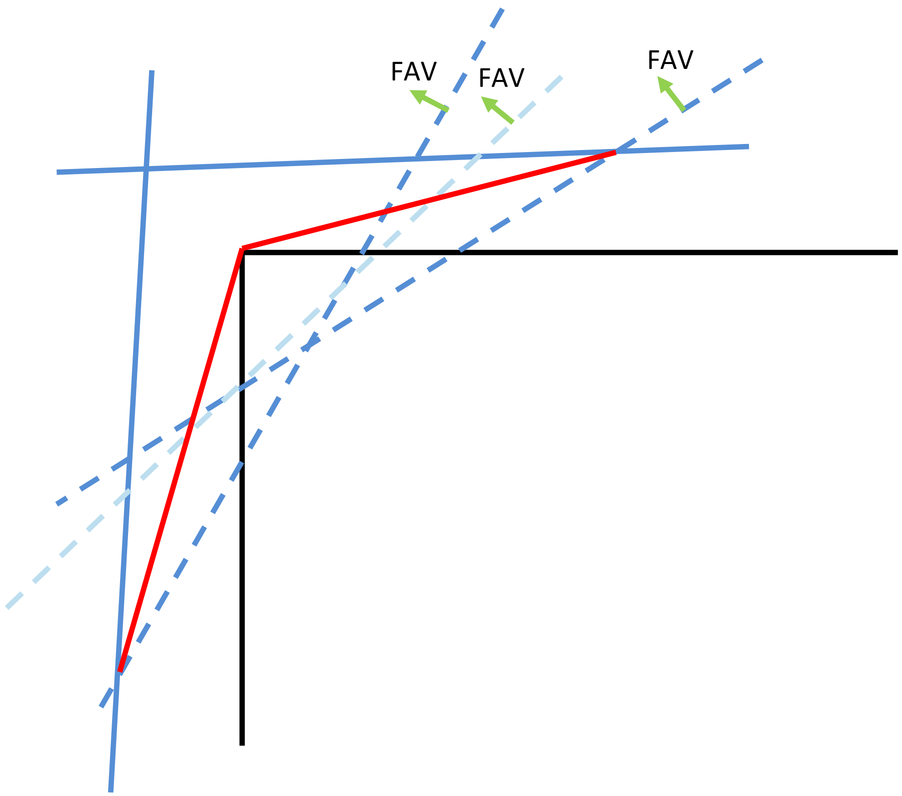


Improved Virtual Branches



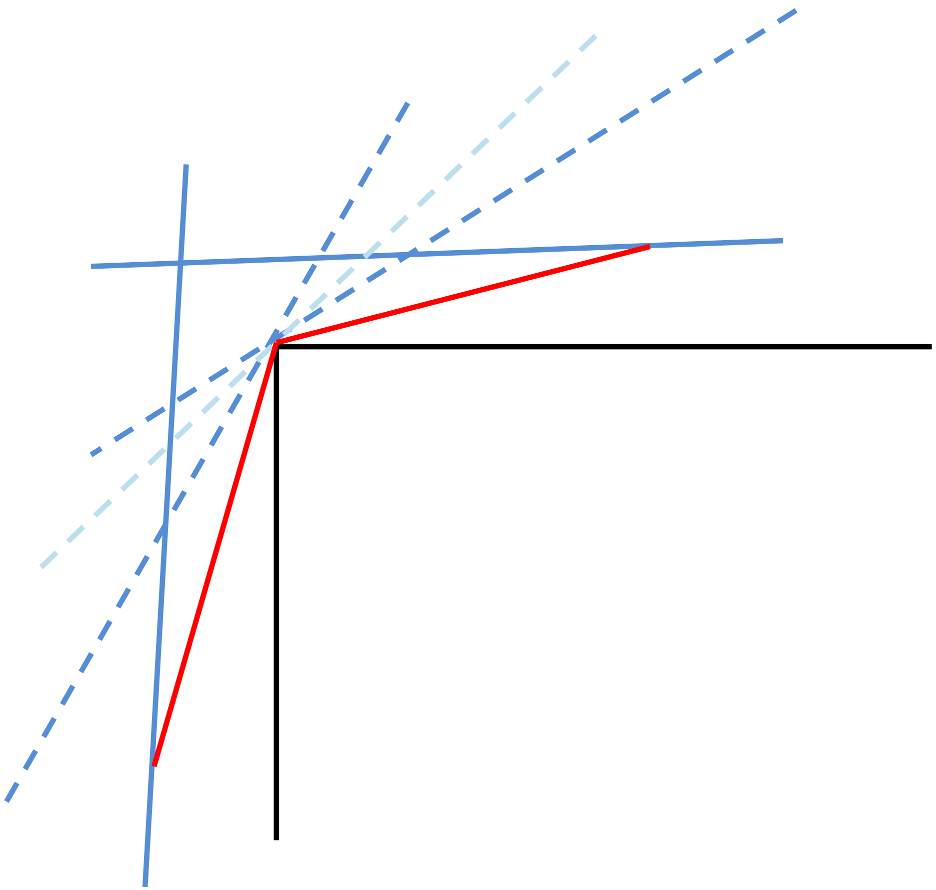
1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull
5. Determine artificial facets of convex hull
6. *For each artificial facet:*
   Add a virtual branch (only once!)

Improved Virtual Branches



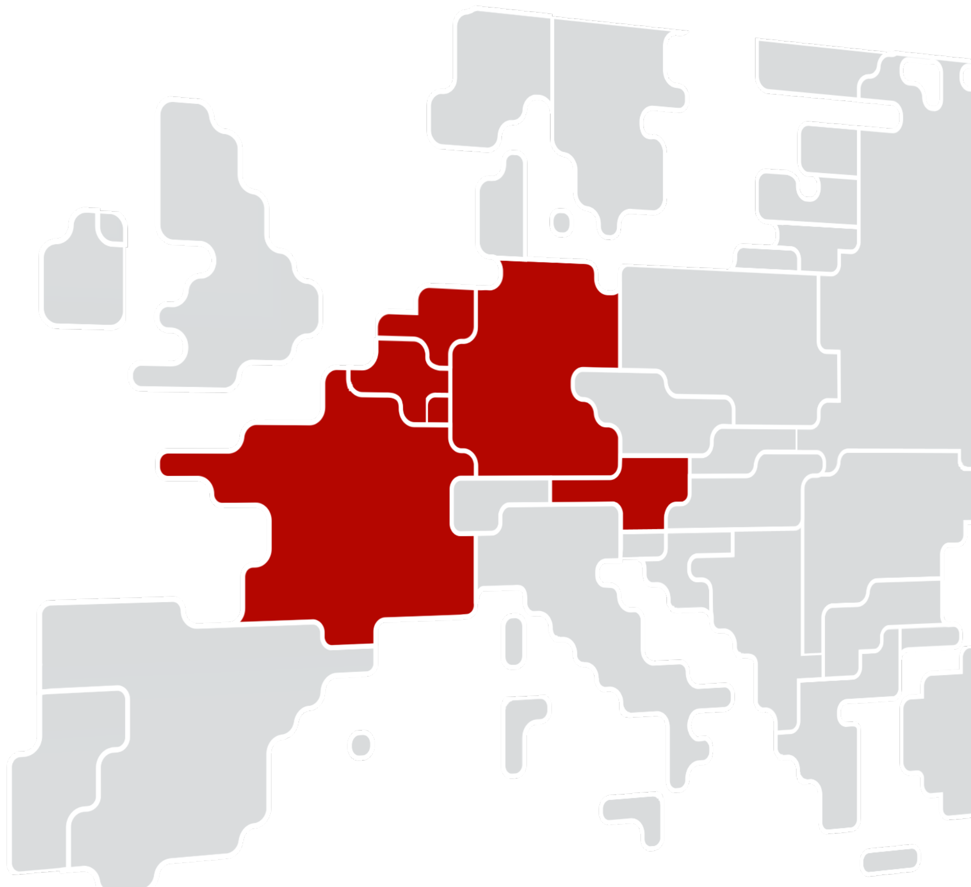
1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull
5. Determine artificial facets of convex hull
6. *For each artificial facet:*
   Add a virtual branch (only once!)
7. *For each overloaded constraint:*
   Apply FAV approach with LTA margin for LTA corner with highest LTA overload

Improved Virtual Branches



1. Determine presolved domain
2. Take vertices of presolved domain
3. Add corners of LTA domain as vertices
4. Only keep vertices of convex hull
5. Determine artificial facets of convex hull
6. *For each artificial facet:*
   Add a virtual branch (only once!)
7. *For each overloaded constraint:*
   Apply FAV approach with LTA margin for LTA corner with highest LTA overload

Annex – Comparison IVB and Extended LTA



Validation of flow-based domain

- Analysis performed:
  - Obtain Min/Max NP from classical Virtual Branch (VB) domain (F206)
  - Obtain Min/Max NP from the Extended LTA inclusion
    - Extended LTA inclusion has been modelled by logarithmo in an optimization prototype
    - F204 virgin domain and LTA domain are used as input domains, and the Min/Max NP are determined by the optimization constrained by the Extended LTA domain formed by these two domains jointly (Balas formulation)
- Key results:
  - **Min/Max NP are almost identical for Extended LTA and VB approach, proving the general applicability of the Extended LTA approach from a CC perspective**
    - Mean deviations of Min/Max NPs are for all bidding zones below 0.02% (< 1 MW)
    - Max deviation of Min/Max NPs is 13MW
      - Even if the deviation is small, it is worth mentioning that this is an extreme outlier for an unlikely market direction, where the VB generation made the FB domain actually too large in the past
    - These slight deviations are due to rounding and small inaccuracies in the creation of Virtual Branches